Answer

TA

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1 Unbiasedness of mu

Want to show that $E(\hat{\mu}) = \mu$.

$\hat{\theta}$ satisfies loglikelihood equation such that $\frac{\partial logL(\theta)}{\partial\theta}|_{\theta=\theta_{\hat{MLE}}} = 0$.

$logL(\mu, \sigma^2) = -\frac{n}{2}log2\pi - \frac{n}{2}log\sigma^2 - \frac{1}{2\sigma^2}\Sigma(X_t - \mu)^2$.

Define $\theta = (\acute{\mu}, \sigma^2)$.

Differentiate log L with respect to μ. Then,

$$\frac{\partial logL}{\partial\mu} = \frac{1}{\sigma^2}\Sigma(X_t - \mu) = 0 \tag{1}$$

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This yields that $\hat{\mu}_{MLE} = \frac{1}{n}\Sigma X_t = \bar{X}$ and thus, since $X \sim N(\mu, \sigma^2)$, $E(\hat{\mu}_{MLE}) = \mu$.